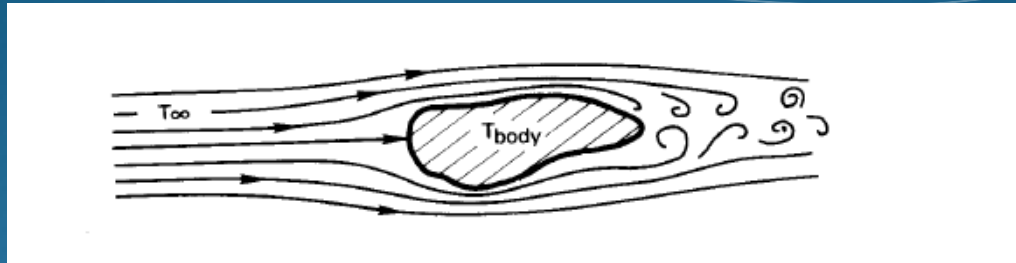


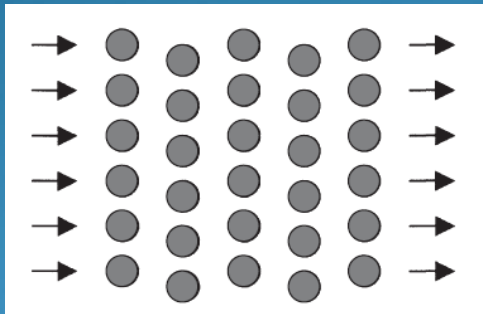
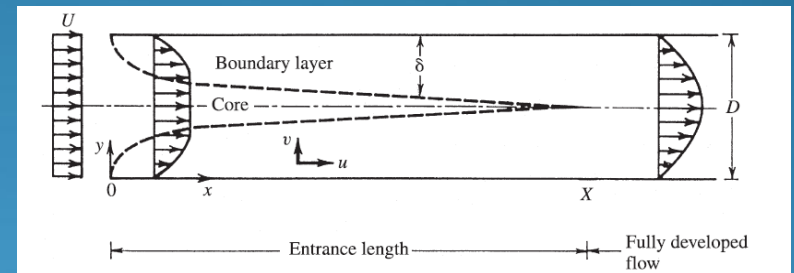
Heat and Mass transfer

Introduction to Convection-1

CONVECTION



Let us consider a typical convective cooling situation. Cool gas flows past a warm body. The fluid immediately adjacent to the body forms a thin slowed-down region called a boundary layer. Heat is conducted into this layer, which sweeps it away, and further downstream, mixes into the stream.



Convective transport from a surface to a fluid in motion, relates the surface to fluid temperature difference and velocity of the fluid, while the fluid does not flow through an enclosure. Such flows are **external flows**.

An **internal flow** is a flow configuration where the flowing fluid is surrounded by solid walls. Streams that flow through ducts are examples of internal flows. Heat exchangers are basically conglomerations of internal flows of various types.

1.1 Reynolds Transport Theorem

The rate of change of N for a system equals the sum of change of N inside any arbitrary control volume in continuum and the rate of efflux of N across the control surface. This can be represented as

$$\frac{DN}{Dt} = \frac{\partial}{\partial t} \int \int \int_V \eta(\rho dV) + \int \int_{A_o} \eta(\rho \mathbf{V} \cdot d\mathbf{A}) \quad (1.1)$$

$N \equiv$ Extensive Property; $\eta \equiv$ specific property

- Mass conservation (Continuity Equation)

$$\frac{DM}{Dt} = \frac{\partial}{\partial t} \int \int \int_V \rho dV + \int \int_{A_o} \rho \mathbf{V} \cdot d\mathbf{A} \quad (1.2)$$

$$\int \int \int_V \frac{\partial \rho}{\partial t} dV + \int \int \int_V \{\nabla \cdot (\rho \mathbf{V})\} dV = 0 \quad (1.3)$$

$$\int \int \int_V \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) \right] dV = 0 \quad (1.4)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (1.5)$$

$$\frac{\partial \rho}{\partial t} + (\mathbf{V} \cdot \nabla) \rho + \rho (\nabla \cdot \mathbf{V}) = 0$$

or,

$$\frac{D\rho}{Dt} + \rho (\nabla \cdot \mathbf{V}) = 0 \quad (\text{General Form}) \quad (1.6)$$

Incompressible flows, $(D\rho/Dt) = 0$ (condition). Therefore, for incompressible flows

$$\rho (\nabla \cdot \mathbf{V}) = 0 \quad (\text{governing equation}) \quad (1.7)$$

or,

$$\nabla \cdot \mathbf{V} = 0 \quad (\text{steady/ unsteady}) \quad (1.8)$$

1.2 Compressible and Incompressible Flows

Let us consider change of volume of a fluid under the action of external forces.

$$E = \frac{-\Delta p}{\Delta \forall / \forall} \quad (1.9)$$

E for water is $2 \times 10^6 \text{ kN/m}^2$, E for air = 101 kN/m^2 . Air is 20,000 times more compressible than water.

The mass continuity may be written as

$$(\forall + \Delta \forall)(\rho + \Delta \rho) = \forall \rho \quad (1.10)$$

which gives

$$\frac{\Delta \forall}{\forall} = -\frac{\Delta \rho}{\rho} \quad (1.11)$$

Now, the modulus of Elasticity, E is expressed as

$$E \approx \frac{-\Delta p}{-(\Delta \rho / \rho)} \approx \frac{\Delta p}{\Delta \rho / \rho} \quad (1.12)$$

or,

$$\frac{\Delta \rho}{\rho} \approx \frac{\Delta p}{E} \approx \frac{1}{2} \frac{\rho V^2}{E} \quad [V \text{ is the velocity}] \quad (1.13)$$

or,

$$\frac{\Delta \rho}{\rho} \sim \frac{1}{2} \frac{V^2}{a^2} \quad (1.14)$$

where, $a = \sqrt{\frac{E}{\rho}}$ = Local acoustic speed
or,

$$\frac{\Delta \rho}{\rho} \sim \frac{1}{2} M^2 \quad (1.15)$$

where, M = Mach number

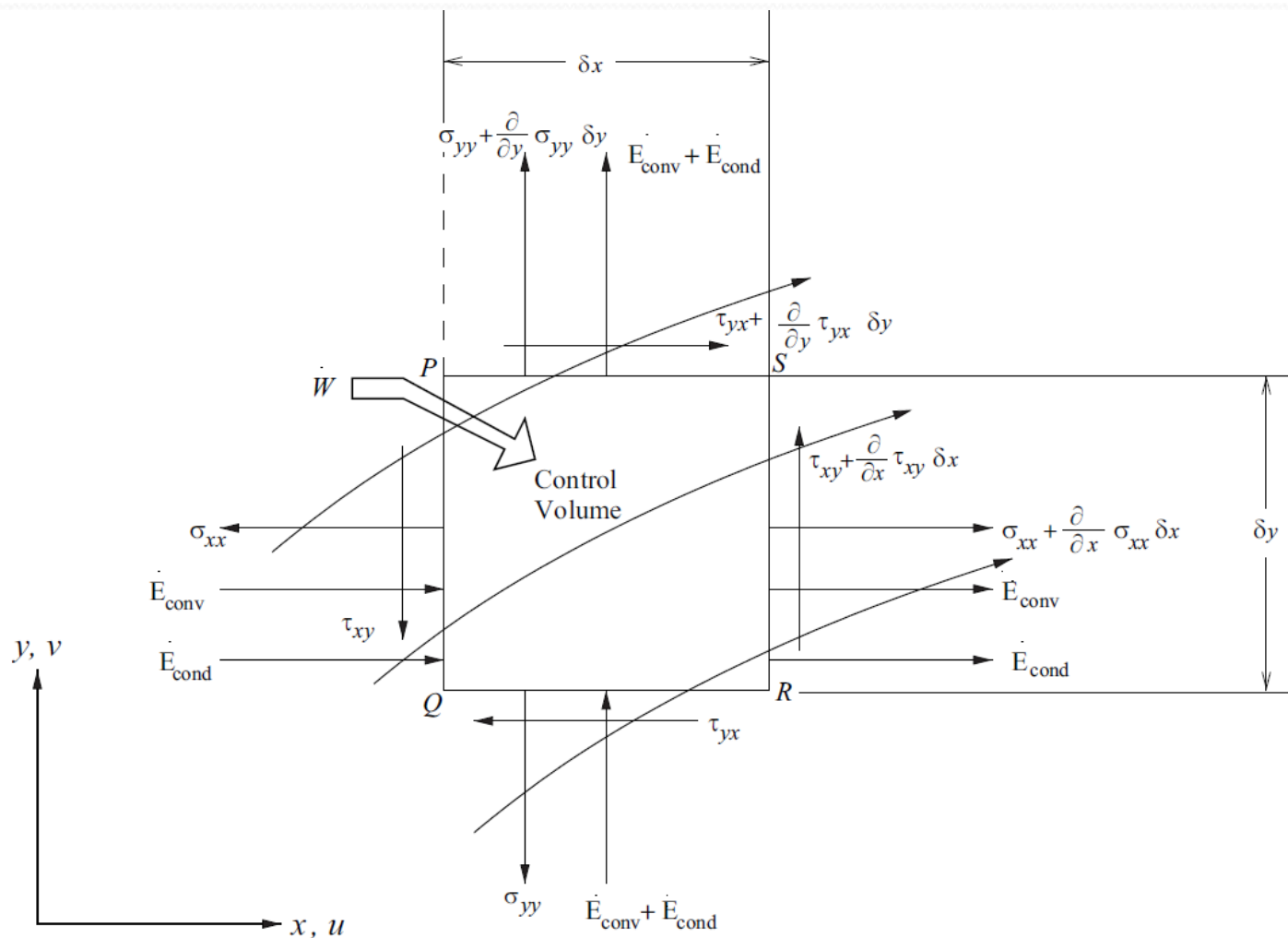
Considering a maximum relative change in density of 5 percent as the criterion of an incompressible flow, the upper limit of Mach number is

$$M \approx 0.334$$

which means, flow of air upto a velocity of 110 m/s under standard condition, can be considered as incompressible flow.

1.3 Energy Equation using Specific Coordinate System

Let us consider a differential control volume (Figure 1.1) in a flow field and account for the energy crossing the control volume (CV) boundary.



Differential Control Volume **PQRS** for Energy Conservation

- Thermal energy per unit mass $\equiv e$
- Kinetic energy per unit mass $= V^2/2$
where $V^2 = u^2 + v^2$

- Rate of change of total energy within the *CV*

$$\frac{\partial}{\partial t} \left\{ \left[\rho \left(e + \frac{V^2}{2} \right) \right] \delta x \delta y \right\} \quad (1.16)$$

rate of mass crossing *PQ* $= \rho u (\delta y.1)$

rate of mass crossing *RS* $= \rho u (\delta y.1) + \frac{\partial}{\partial x} [\rho u (\delta y.1)] \delta x$

rate of energy crossing *PQ* $= \rho u \delta y \left(e + \frac{V^2}{2} \right)$

rate of energy crossing *RS*

$$\left\{ \left(e + \frac{V^2}{2} \right) \rho u + \frac{\partial}{\partial x} \left[\left(e + \frac{V^2}{2} \right) \rho u \right] \delta x \right\} (\delta y.1) \quad (1.17)$$

Net convective efflux of energy across this pair of faces (*PQ* and *RS*)

$$\frac{\partial}{\partial x} \left[\left(e + \frac{V^2}{2} \right) \rho u \right] \delta x \delta y \quad (1.18)$$

Similarly, we can find the efflux across the faces *PS* and *QR*.

- efflux (both in x and y direction) of energy convected:

$$\left\{ \frac{\partial}{\partial x} \left[\left(e + \frac{V^2}{2} \right) \rho u \right] + \frac{\partial}{\partial y} \left[\left(e + \frac{V^2}{2} \right) \rho v \right] \right\} \delta x \delta y \quad (1.19)$$

Next, we shall consider diffusive transport of energy. Conduction (diffusive) flux across PQ into the control volume is given by

$$-k \frac{\partial T}{\partial x} (\delta y.1) \quad (1.20)$$

Conduction flux across RS=

$$-k \frac{\partial T}{\partial x} \delta y + \left\{ \frac{\partial}{\partial x} \left[-k \frac{\partial T}{\partial x} \right] \delta x \right\} \delta y \quad (1.21)$$

- net conduction flux in x direction,

$$-\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) \delta x \delta y \quad (1.22)$$

Similarly, net conduction flux across the whole control volume

$$-\left[\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) \right] \delta x \delta y \quad (1.23)$$

Total efflux of energy is now:

$$\left[\frac{\partial}{\partial x} \left\{ \left(e + \frac{V^2}{2} \right) \rho u \right\} + \frac{\partial}{\partial y} \left\{ \left(e + \frac{V^2}{2} \right) \rho v \right\} \right] \delta x \delta y - \left[\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) \right] \delta x \delta y \quad (1.24)$$

In vector notation we can write that efflux of energy through the Control Surface (CS)

$$\left[\nabla \cdot \left\{ \rho \vec{V} \left(e + \frac{V^2}{2} \right) \right\} - \nabla \cdot (k \nabla T) \right] \delta x \delta y \quad (1.25)$$

The Reynolds transport theorem can be stated as

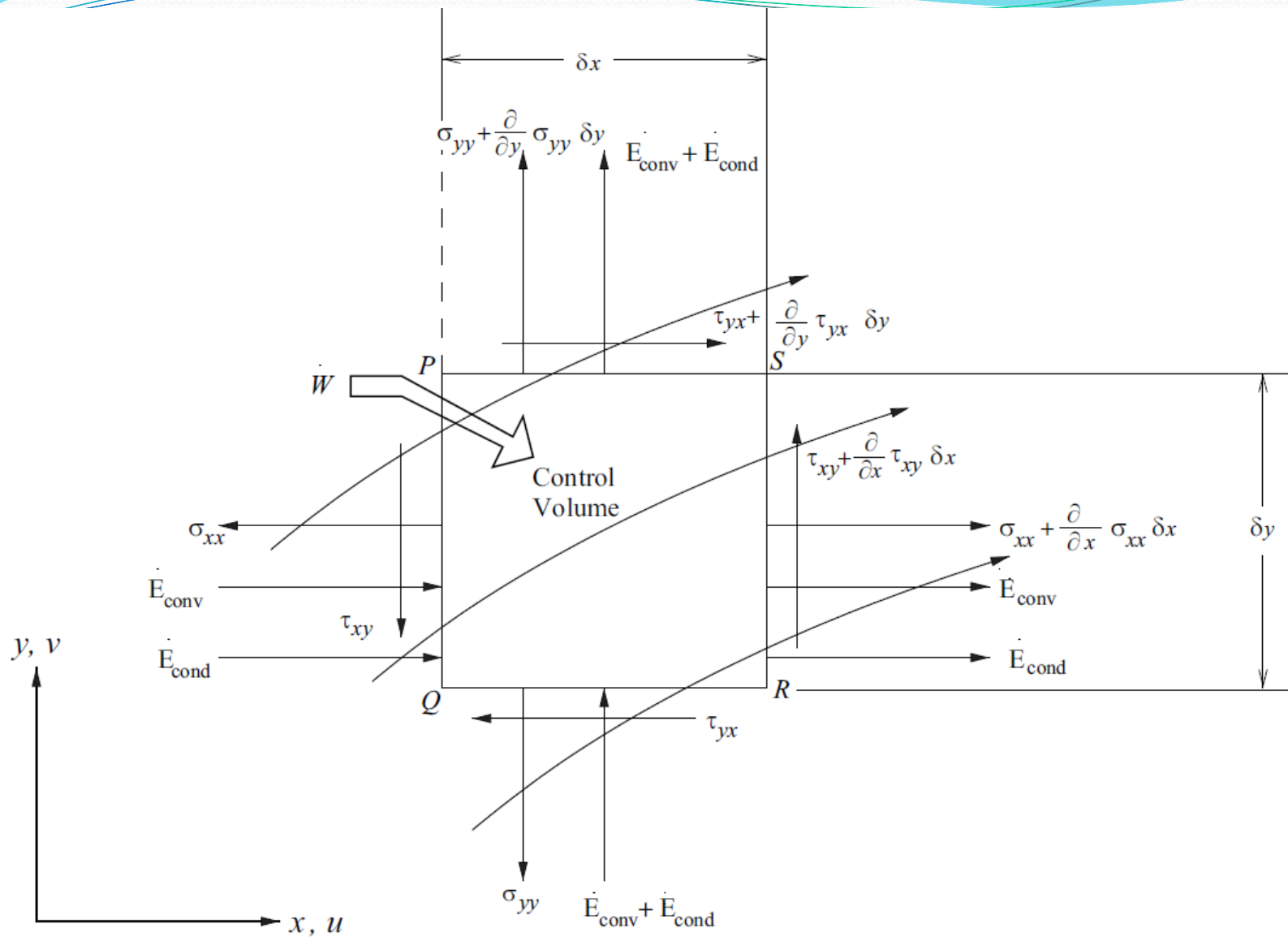
Rate of change of energy for the system = rate of change of energy in the Control Volume (CV) + rate of efflux of energy through the Control Surface (CS) of the CV .

According to the First law of Thermodynamics:

Rate of energy production within the CV + rate of work done on the CV = Rate of change of energy for the system.

Let us evaluate the quantity, rate of work done on the CV. The rate at which body force $\vec{f}$ does work is given by force time velocity

$$\rho [f_x u + f_y v] \delta x \delta y = \rho \vec{f} \cdot \vec{V} \delta x \delta y \quad (1.26)$$



Work done (rate) by the surface forces on the PQ plane of CV :

$$-\sigma_{xx}u\delta y - \tau_{xy}v\delta y \quad (1.27)$$

Work done (rate) by the surface forces on RS plane is :

$$\left[(\sigma_{xx}u) + \frac{\partial}{\partial x} (\sigma_{xx}u) \delta x + \tau_{xy}v + \frac{\partial}{\partial x} (\tau_{xy}v) \delta x \right] \delta y \quad (1.28)$$

Net work done (rate) by the surface forces in x direction

$$\left[\frac{\partial}{\partial x} (\sigma_{xx}u) + \frac{\partial}{\partial x} (\tau_{xy}v) \right] \delta x \delta y \quad (1.29)$$

Similarly, we can determine the net work done (rate) by the surface forces in y direction, and adding these two we get work done (rate) on the CV:

$$\left[\frac{\partial}{\partial x} (\sigma_{xx}u) + \frac{\partial}{\partial y} (\sigma_{yy}v) + \frac{\partial}{\partial x} (\tau_{xy}v) + \frac{\partial}{\partial y} (\tau_{yx}u) \right] \delta x \delta y \quad (1.30)$$

We assume the sources of heat such that the strength per unit volume is $\dot{S}_{th}$, the energy production within the CV = $\dot{S}_{th}\delta x\delta y$

The conservation of energy requires (invoking Eqns (1.16), (1.24), (1.26) and (1.30))

$$\begin{aligned}
 & \frac{\partial}{\partial t} \left\{ \rho \left(e + \frac{V^2}{2} \right) \right\} + \frac{\partial}{\partial x} \left\{ \rho u \left(e + \frac{V^2}{2} \right) \right\} \\
 & + \frac{\partial}{\partial y} \left\{ \rho v \left(e + \frac{V^2}{2} \right) \right\} - \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) \\
 & = \rho \{ f_x u + f_y v \} + \left[\frac{\partial}{\partial x} \{ \sigma_{xx} u + \tau_{xy} v \} + \frac{\partial}{\partial y} \{ \sigma_{yy} v + \tau_{yx} u \} \right] + \dot{S}_{th} \quad (1.31)
 \end{aligned}$$

In order to simplify these terms, let us take a recourse to the mechanical energy part through the NS equation. Multiplying the x momentum equation by u and y momentum equation by v and adding, we get

$$\begin{aligned}
 & \rho \left[\frac{\partial}{\partial t} \left(\frac{V^2}{2} \right) + u^2 \frac{\partial u}{\partial x} + uv \frac{\partial u}{\partial y} + uv \frac{\partial v}{\partial x} + v^2 \frac{\partial v}{\partial y} \right] \\
 & = \rho \{ f_x u + f_y v \} + u \left\{ \frac{\partial}{\partial x} \sigma_{xx} + \frac{\partial}{\partial y} \tau_{xy} \right\} + v \left\{ \frac{\partial}{\partial x} \tau_{yx} + \frac{\partial}{\partial y} \sigma_{yy} \right\} \quad (1.32)
 \end{aligned}$$

First two convective terms of the LHS of Eqn. (1.31) can be rearranged as

$$\begin{aligned}
 & \frac{\partial}{\partial x} \left\{ \rho u \left(e + \frac{V^2}{2} \right) \right\} + \frac{\partial}{\partial y} \left\{ \rho v \left(e + \frac{V^2}{2} \right) \right\} = \rho u \frac{\partial e}{\partial x} + e \frac{\partial}{\partial x} (\rho u) + \frac{V^2}{2} \frac{\partial}{\partial x} (\rho u) \\
 & + \rho u \frac{\partial}{\partial x} \left(\frac{V^2}{2} \right) + \rho v \frac{\partial e}{\partial y} + e \frac{\partial}{\partial y} (\rho v) + \frac{V^2}{2} \frac{\partial}{\partial y} (\rho v) + \rho v \frac{\partial}{\partial y} \left(\frac{V^2}{2} \right) \quad (1.33)
 \end{aligned}$$

The temporal term of (1.31)

$$\frac{\partial}{\partial t} \left\{ \rho \left(e + \frac{V^2}{2} \right) \right\} = \rho \frac{\partial e}{\partial t} + e \frac{\partial \rho}{\partial t} + \rho \frac{\partial}{\partial t} \left(\frac{V^2}{2} \right) + \frac{V^2}{2} \left(\frac{\partial \rho}{\partial t} \right) \quad (1.34)$$

First three terms of Eqn. (1.31) will yield

$$\begin{aligned} & \rho \left(u \frac{\partial e}{\partial x} + v \frac{\partial e}{\partial y} + \frac{\partial e}{\partial t} \right) + e \left(\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) \right) + \frac{V^2}{2} \left(\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) \right) \\ & + \rho \frac{\partial}{\partial t} \left(\frac{V^2}{2} \right) + \left[\rho \left\{ u \frac{(2u \frac{\partial u}{\partial x} + 2v \frac{\partial v}{\partial x})}{2} \right\} + \rho \left\{ v \frac{(2u \frac{\partial u}{\partial y} + 2v \frac{\partial v}{\partial y})}{2} \right\} \right] \\ & = \rho \left[\frac{\partial}{\partial t} \left(\frac{V^2}{2} \right) + u^2 \frac{\partial u}{\partial x} + uv \frac{\partial u}{\partial y} + uv \frac{\partial v}{\partial x} + v^2 \frac{\partial v}{\partial y} \right] + \rho \left[\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial x} + v \frac{\partial e}{\partial y} \right] \quad (1.35) \end{aligned}$$

Invoking this and considering homogeneous isotropic medium, the energy equation (1.31) can be written as

$$\begin{aligned} & \rho \left[\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial x} + v \frac{\partial e}{\partial y} \right] + \rho \left[\frac{\partial}{\partial t} \left(\frac{V^2}{2} \right) + u^2 \frac{\partial u}{\partial x} + uv \frac{\partial u}{\partial y} + uv \frac{\partial v}{\partial x} + v^2 \frac{\partial v}{\partial y} \right] \\ & + \left(-k \frac{\partial^2 T}{\partial x^2} - k \frac{\partial^2 T}{\partial y^2} \right) = \rho \{ f_x u + f_y v \} + \sigma_{xx} \frac{\partial u}{\partial x} + u \frac{\partial}{\partial x} \sigma_{xx} + \tau_{xy} \frac{\partial v}{\partial x} \\ & + v \frac{\partial}{\partial x} \tau_{xy} + \sigma_{yy} \frac{\partial v}{\partial y} + v \frac{\partial}{\partial y} \sigma_{yy} + \tau_{yx} \frac{\partial u}{\partial y} + u \frac{\partial}{\partial y} \tau_{yx} + \dot{S}_{th} \quad (1.36) \end{aligned}$$

Subtracting Eqn. (1.32) from Eqn. (1.36) and considering the stress tensor symmetric we get

$$\begin{aligned} & \rho \left[\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial x} + v \frac{\partial e}{\partial y} \right] - k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \\ &= \left\{ \sigma_{xx} \frac{\partial u}{\partial x} + \tau_{xy} \frac{\partial v}{\partial x} + \sigma_{yy} \frac{\partial v}{\partial y} + \tau_{yx} \frac{\partial u}{\partial y} \right\} + \dot{S}_{th} \end{aligned} \quad (1.37)$$

Let us look at terms within braces { }

$$\begin{aligned} & \frac{\partial u}{\partial x} \sigma_{xx} = \left(-p + 2\mu \frac{\partial u}{\partial x} - \frac{2}{3}\mu \left\{ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right\} \right) \frac{\partial u}{\partial x} \\ &= -p \frac{\partial u}{\partial x} + 2\mu \left(\frac{\partial u}{\partial x} \right)^2 - \frac{2}{3}\mu \left(\frac{\partial u}{\partial x} \right)^2 - \frac{2}{3}\mu \left(\frac{\partial u}{\partial x} \right) \left(\frac{\partial v}{\partial y} \right) \end{aligned} \quad (1.38)$$

$$\begin{aligned} & \frac{\partial v}{\partial y} \sigma_{yy} = \left(-p + 2\mu \frac{\partial v}{\partial y} - \frac{2}{3}\mu \left\{ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right\} \right) \frac{\partial v}{\partial y} \\ &= -p \frac{\partial v}{\partial y} + 2\mu \left(\frac{\partial v}{\partial y} \right)^2 - \frac{2}{3}\mu \left(\frac{\partial v}{\partial y} \right)^2 - \frac{2}{3}\mu \left(\frac{\partial u}{\partial x} \right) \left(\frac{\partial v}{\partial y} \right) \end{aligned} \quad (1.39)$$

$$\tau_{xy} \frac{\partial v}{\partial x} = \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \left(\frac{\partial v}{\partial x} \right) = \mu \left(\frac{\partial v}{\partial x} \right)^2 + \mu \left(\frac{\partial v}{\partial x} \cdot \frac{\partial u}{\partial y} \right) \quad (1.40)$$

$$\tau_{yx} \frac{\partial u}{\partial y} = \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \left(\frac{\partial u}{\partial y} \right) = \mu \left(\frac{\partial u}{\partial y} \right)^2 + \mu \left(\frac{\partial v}{\partial x} \cdot \frac{\partial u}{\partial y} \right) \quad (1.41)$$

Equation (1.37) may now be written as

$$\begin{aligned} \rho \left[\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial x} + v \frac{\partial e}{\partial y} \right] &= k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] - p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \\ &+ \underbrace{\mu \left[2 \left\{ \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right\} + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right]}_{\text{Viscous dissipation} \equiv \mu \Phi} + \dot{S}_{th} \end{aligned} \quad (1.42)$$

Equation (1.42) is the general form of energy equation, valid for both the compressible and incompressible flows.

Now, let us consider enthalpy per unit mass = $i = e + \frac{p}{\rho}$

Substituting i in equation(1.42), we get:

$$\begin{aligned} \rho \left[\frac{\partial i}{\partial t} + u \frac{\partial i}{\partial x} + v \frac{\partial i}{\partial y} \right] &- \rho \left[\frac{\partial}{\partial t} \left(\frac{p}{\rho} \right) + u \frac{\partial}{\partial x} \left(\frac{p}{\rho} \right) + v \frac{\partial}{\partial y} \left(\frac{p}{\rho} \right) \right] \\ &= k \nabla^2 T - p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \mu \Phi + \dot{S}_{th} \end{aligned} \quad (1.43)$$

or,

$$\rho \left[\frac{\partial i}{\partial t} + u \frac{\partial i}{\partial x} + v \frac{\partial i}{\partial y} \right] - \left[\frac{\partial p}{\partial t} - \frac{p}{\rho} \frac{\partial \rho}{\partial t} + u \frac{\partial p}{\partial x} - \frac{up}{\rho} \frac{\partial \rho}{\partial x} + v \frac{\partial p}{\partial y} - \frac{vp}{\rho} \frac{\partial \rho}{\partial y} \right] = k \nabla^2 T - p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \mu \Phi + \dot{S}_{th} \quad (1.44)$$

or,

$$\rho \left[\frac{\partial i}{\partial t} + u \frac{\partial i}{\partial x} + v \frac{\partial i}{\partial y} \right] = k \nabla^2 T + \underbrace{\left(\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} \right)}_{\frac{\partial p}{\partial t} + \vec{V} \cdot \nabla(p)} - \frac{p}{\rho} \left[\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} \right] + \mu \Phi + \dot{S}_{th} \quad (1.45)$$

or,

$$\rho \left[\frac{\partial i}{\partial t} + u \frac{\partial i}{\partial x} + v \frac{\partial i}{\partial y} \right] = k \nabla^2 T + \frac{Dp}{Dt} - \frac{p}{\rho} \left[\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) \right] + \mu \Phi + \dot{S}_{th} \quad (1.46)$$

or,

$$\rho \left[\frac{\partial i}{\partial t} + \vec{V} \cdot \nabla(i) \right] = k \nabla^2 T + \frac{Dp}{Dt} + \mu \Phi + \dot{S}_{th} \quad (1.47)$$

or,

$$\rho \frac{Di}{Dt} = k \nabla^2 T + \frac{Dp}{Dt} + \mu \Phi + \dot{S}_{th} \quad (1.48)$$

Substituting, $i = c_p T$

$$\rho c_p \frac{DT}{Dt} = \frac{Dp}{Dt} + k \nabla^2 T + \mu \Phi + \dot{S}_{th} \quad (1.49)$$

This is the general form of energy equation.

Special Case: For liquids, we can directly write from (1.42)

$$\rho c \frac{DT}{Dt} = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \mu \Phi + \dot{S}_{th} \quad (1.50)$$

For liquids, $c_p = c_v = c$ and $p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0$. In the case of liquids, the viscous dissipation term will have to be modified by invoking the incompressibility condition.



Thank you